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STABILITY OF VISCO-ELASTIC(RIVLIN-ERICKSEN) COMPRESSIBLE FLUID IN POROUS MEDIUM : EFFECT OF MAGNETIC FIELD

Preeti Ahluwaliya*, Aftab Alam, Sudhir Kumar
* Department of Mathematics, H L M College ,Ghaziabad,U.P.(INDIA)
Department of Mathematics, SRM University, NCR, Campus Ghaziabad, U.P.(INDIA)
Department of Mathematics, S.D(P.G.)College , Muzaffarnagar,U.P.-251001(INDIA)

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ABSTRACT

The hydro-magnetic instability of Rivlin-Ericksen compressible fluid in porous medium is considered by applying normal mode technique, the dispersion result is obtained. The important result obtained in this present paper, by a number of theorems providing conditions for stability or instability, and the case of slightly compressible fluid neutral or unstable modes are non-oscillatory and oscillatory mode is also stable. It is also found that the system are stable, unstable or neutral analysis for subsonic and supersonic disturbances with weak applying magnetic field (i.e.S<< 1**)** and another bounds on the complex wave velocity of unstable modes for $N^2 < 0$.

KEYWORDS: Hydromagnetic instability , Rivlin-Ericksen, compressible fluid , porous medium,weak magnetic field.

INTRODUCTION

In classical hydrodynamics, the majority of the previously known results related to the study of stability of fluid flows concerns either compressible fluid in the absence of porous medium of incompressible fluid through a porous medium. However, in many flows of engineering interest, fluid speeds exceed the speed of sound and density changes can be quite large. These flows are called compressible flows. It seems that the effect of Darcy resistance in combination with compressibility, to the best of our knowledge, is almost uninvestigated so far. However, since the compressibility is exhibited by all fluids in appropriate circumstances, it is necessary to include its effect into the stability analysis of a system in the presence of a porous medium.

Hydromagnetic stability of gravitationally stratified dusty fluid rotated in a porous medium discussed by Rachna and Jaimala **[1]**. It is found that in this paper include the equivalence of two and three dimensional disturbances, sufficient of stability estimates on the growth rate of unstable modes and the existence of variational principle. Kumar and Singh **[2]** have studied the stability of the plane interface separating two visco-elastic (Rivlin-Ericksen) superposed fluids in he presence of suspended particles. The stability analysis has been carried out, for mathematical simplicity for two highly visco-elastic fluids of equal kinematic viscosities and equal kinematic visco-elasticities. The system is found to be stable for stable configuration and unstable for unstable configuration. Sharma and Kango **[3]** have studied the thermal convection in Rivlin-Ericksen elastico-viscous fluid in porous medium in the presence of uniform magnetic field. Prakash and Kumar **[4]** studied the thermal instability in Rivlin-Ericksen elastico-viscous fluid in the presence of larmar radius and variable gravity in porous medium. Sharma and Rana **[5]** studied the magnetogravitational instability of a thermally conducting rotating Rivlin-Ericksen fluid through a porous medium with finite conducting in the presence of Hall current. The wave propagation has been considered for both parallel and perpendicular axes of rotation and the magnetic field is being taken in the vertical direction.

Recently, Kim et al. **[6]** investigated by the thermal instability of Visco-Elastic fluid in porous medium using the modified Darcy-Oldroyd model. Kumar et al. **[7]** studied the thermal instability of Walter's B' Visco-Elastic fluid permeated with suspended particles in hydromagnetics in porous medium. Jaimala and Agrawal **[8]** have discussed the stability of a density stratified fluid with horizontal streaming through a porous medium. Sharma and Kumar **[9]** have discussed the thermal instability of a layer of Rivlin-Ericksen elastico-viscous fluid acted on by a uniform rotation and found that rotation has a stabilizing effect and introduce oscillatory modes in the system. Pundir and Pundir **[10]** have discussed the thermal instability of a continuously stratified Visco-Elastic (Oldroydian) fluid in porous medium in the presence of a horizontal magnetic field.

Pal **[11]** discussed the Rayleigh-Taylor instability of Rivlin-Erickson plasma in presence of a variable magnetic field and suspended particles in porous medium. Prakash and Chand **[12]** have discussed the thermal instability of Oldroydian visco-elastic fluid in the presence of finite larmor radius, rotation and variable gravity field in porous medium. Agrawal et al. **[13]** have studied the shear flow instability of visco-elastic fluid in anisotropic porous medium. Kumar et al. **[14]** investigated by the thermal instability of a rotating (Rivlin-Ericksen) visco-elastic fluid in the presence of uniform vertical magnetic field. It is found that rotation has a stabilizing effect whereas the magnetic field has both stabilizing and destabilizing effects. Bahadur **[15]** has discussed by the thermal instability of (Rivlin-Ericksen) elastico-viscous rotating fluids in presence of suspended particles in porous medium.

With the growing importance of non-Newtonian fluid in modern technology and industries, the investigations on such fluids are desirable. There are many visco-elastic fluids that can't be characterized by Maxwell's constitutive relations or Oldroyd's constitutive relations. One such class of Visco-Elastic is the Rivlin-Ericksen fluid. This and other class of polymers, ropes, cushions, seats, foams, plastic, engineering equipments, etc. Recently polymers are also used in agriculture, communication appliances and in biomedical applications. When fluid permeates a porous material the gross effect is represented by the Darcy's law. As a result of this macroscopic law, the usual viscous is replaced by $\left[-\frac{1}{k_1}\left(\mu+\mu'\frac{\partial}{\partial t}\right)\mathbf{q}\right]$, where μ and μ' are the viscosity and viscoelasticity of the thermosolutal instability of Rivlin-Ericksen fluid, k_1 is the medium permeability and $\mathbf{q}$ is the Darcian (filter) velocity. However, hydromagnetic instability of Rivlin-Ericksen compressible fluid in porous media seems to the best of our knowledge uninvestigated so far. In this chapter, therefore, we have made an attempt to critically examine the hydromagnetic instability of Rivlin-Ericksen compressible fluid in porous medium. It can be looked upon an extension of hydromagnetic stability of a compressible fluid in a porous medium discussed by Jaimala **[16]**.

EQUATIONS OF MOTION

In the equations of motion for the gas, the presence of particles adds an extra force term proportional to the velocity difference between particles and gas. Assuming that the usual viscous dissipation along with the dissipation due to Darcy resistance is present, the governing equations for the gas can be written as

$$\nabla.\mathbf{q} = 0, \quad \textbf{(1)}$$

$$\frac{\rho}{\phi}\frac{\partial \mathbf{q}}{\partial t} + \frac{\rho}{\phi^2}(\mathbf{q}.\nabla)\mathbf{q} = -\nabla p - \frac{\rho}{k_1}\left(\nu + \nu'\frac{\partial}{\partial t}\right)\mathbf{q} + \frac{1}{\mu_0}(\nabla\times\mathbf{H})\times\mathbf{H} - \mathbf{g}\rho\boldsymbol{\lambda}, \quad \textbf{(2)}$$

$$\frac{\partial \rho}{\partial t} + \frac{1}{\phi}\rho\nabla.\mathbf{q} = 0, \quad \textbf{(3)}$$

$$\frac{DP}{Dt} = \left(\frac{\partial P}{\partial \rho}\right)_s \frac{D\rho}{Dt} = c_0^2\frac{D\rho}{Dt} = -\frac{1}{\phi}c_0^2\rho\nabla.\mathbf{q} \quad [\text{using } \mathbf{3}] \quad \textbf{(4)}$$

$$\frac{D\mathbf{H}}{Dt} = \frac{1}{\phi}(\mathbf{H}.\nabla)\mathbf{q} - \frac{1}{\phi}(\nabla.\mathbf{q})\mathbf{H} \quad \textbf{(5)}$$

and $$\nabla.\mathbf{H} = 0, \quad \textbf{(6)}$$

where, $\mathbf{H}$, s and μ_0 are respectively the magnetic field vector, entropy per unit mass, magnetic permeability of the vacuum. The operator $\frac{D}{Dt}$ is defined as

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \frac{1}{\phi}\mathbf{U}.\nabla \quad \textbf{(7)}$$

and other parameters have their usual meaning. The equations from **(1)** to **(6)** represent respectively, the non-linearity equation, the momentum equation, the equation of mass conservation, the energy equation, the induction equation and divergence free condition for magnetic field. In the equation **(2)**, third term or right hand side represents the Lorenze force. Finally c_0^2 in energy equation **(4)** is the square of the sound speed (i.e., the velocity of propagation of small disturbances) in the medium, depends upon z coordinate only and defined as

$$c_0^2 = \left(\frac{\partial p}{\partial \rho}\right)_s \tag{8}$$

Following the usual procedure and considering the perturbations

$$\left.\begin{aligned} &\mathbf{q} = (0, 0, 0), \\ &\mathbf{H} = [H_0(uniform), 0, 0], \\ &\rho = \rho(z) \\ \text{and} \quad &p = p(z), \end{aligned}\right\} \tag{9}$$

We get
and, we assume that the perturbations can be expanded into normal modes so that the dependence on x, z and t of a general disturbance $f'(x,z,t)$ is taken as

$$f'(x,z,t) = f(z)\exp\left[ik\left(x - \frac{c}{\phi}t\right)\right] \tag{10}$$

where $k > 0$ represents the wave number in x-direction and c, in general, is complex
we get the final governing equation

$$\begin{aligned} k^2\rho\left[c^2 + ik^{-1}R_D^{-1}c + k_1^{-1}R_{ve}^{-1}c^2 - S\right]w = D\Bigg\{\rho\left(\frac{c^2 + ik^{-1}R_D^{-1}c + k_1^{-1}R_{ve}^{-1}c^2}{\Delta} - S\right)Dw \\ - \frac{g\rho}{c_0^2\Delta}\left(c^2 + ik^{-1}R_D^{-1}c + k_1^{-1}R_{ve}^{-1}c^2\right)w\Bigg\} \\ + \frac{g\rho}{c_0^2\Delta}\left(c^2 + ik^{-1}R_D^{-1}c + k_1^{-1}R_{ve}^{-1}c^2\right)\left(D - \frac{g}{c_0^2}\right)w + N^2\rho w \end{aligned} \tag{11}$$

where, $N^2 = -g\left(\frac{g}{c_0^2} + \frac{D\rho}{\rho}\right)$ is the square of the Väisälä Brunt frequency,

$S = \frac{H_0^2\phi^2}{\mu_0\rho U_0^2}$ is the magnetic force number and is a function of z because of the presence of ρ,

$R_D^{-1} = \frac{v\phi^2 d}{k_1 U_0}$ is the inverse of the Darcy-Reynolds number

$R_{ve}^{-1} = \frac{\phi v'}{d^2}$

and $\Delta = 1 - \frac{c^2 + ik^{-1}R_D^{-1}c + k_1^{-1}R_{ve}^{-1}c^2}{c_0^2}$.

The boundary conditions now become

$$w = 0 \text{ at } z = 0 \text{ and } z = 1. \tag{12}$$

In the limit $R_D \to \infty$ (i.e., in the absence of porous medium) equation **(11)** reduces to the one obtained by Agrawal and Rastogi **[17]**. Here also, the character of the problem is changed due to the presence of porous medium and the system is stable, neutral or unstable according as $c_i <, =$ or > 0.

Because of the complicated character of the problem, it is extremely difficult to discuss the problem in its full generality. Therefore, a number of physically important cases have been dealt with in the subsequent sections.

THE CASE OF A SLIGHTLY COMPRESSIBLE FLUID

The compressibility of a fluid is the inverse of its bulk modulus of elasticity. Therefore, the compressibility of the fluid is the ratio of the relative change of the volume to the change in applied pressure. Thus fluids which require a large pressure change to density, are called slightly compressible fluids, while fluids for which even a small pressure change makes an appreciable change in the density are called highly compressible or to say, simply compressible fluids.

It is well known that the gases are highly compressible while the liquids are slightly compressible. Further, it is evident that in highly compressible fluids, the velocity of sound will be small while in slightly compressible fluids, it will be large. Therefore, it is very important, in the context of many physical situations to investigate extensively, as we have done here, the case of slightly compressible fluids.

For slightly compressible fluids, g/c_0^2 is assumed to be small and of the order of perturbations so that the term $\left(g/c_0^2\right)w$ being of second order in perturbations can be ignored in comparison to w or its derivatives. Under this approximation, we get the reduced stability governing equation as

$$k^2\rho\left(c^2+ik^{-1}R_D^{-1}c+k_1^{-1}R_{ve}^{-1}c^2-S\right)w=D\left\{\rho\left(\frac{c^2+ik^{-1}R_D^{-1}c+k_1^{-1}R_{ve}^{-1}c^2}{\Delta}-S\right)Dw\right\}+N^2\rho w \cdot$$

(13)

In obtaining equation **(13)**, we have further assumed that $\frac{g}{c_0^2}=\left|\frac{D\rho}{\rho}\right|$ as k is small.

Multiplying equation **(13)** by w^*, the complex conjugate of w, integrating over the range of z and using the boundary conditions, we get after some arrangement of terms

$$\int \rho\left(\frac{c^2+ik^{-1}R_D^{-1}c+k_1^{-1}R_{ve}^{-1}c^2}{\Delta}-S\right)|Dw|^2\,dz+k^2\int \rho\left(c^2+ik^{-1}R_D^{-1}c+k_1^{-1}R_{ve}^{-1}c^2-S\right)|w|^2\,dz$$

$$-\int N^2\rho|w|^2\,dz=0\cdot \quad \mathbf{(14)}$$

The real and imaginary part of this equation are given by

$$\int \rho\Big[\left\{\left(c_r^2-c_i^2\right)(1+k_1^{-1}R_{ve}^{-1})-\frac{(1+k_1^{-1}R_{ve}^{-1})(c_i|c|^2 .R_D^{-1}k^{-1}+(1+k_1^{-1}R_{ve}^{-1})|c|^4)}{c_0^2}\right.$$

$$\left.-k^{-1}R_D^{-1}\left(c_i+\frac{R_D^{-1}k^{-1}|c|^2+(1+k_1^{-1}R_{ve}^{-1})c_i|c|^2}{c_0^2}\right)\right\}\frac{1}{|\Delta|^2}-S\ \Big]|Dw|^2\,dz \quad \mathbf{(15)}$$

and

$$c_r\left\{2c_i(1+k_1^{-1}R_{ve}^{-1})+k^{-1}R_D^{-1}\right\}\int \rho\left(\frac{|Dw|^2}{|\Delta|^2}+k^2|w|^2\right)dz=0\cdot \quad \mathbf{(16)}$$

We now prove the following theorems.

Theorem 3.1 : Neutral or unstable modes are non-oscillatory.

Proof : Equation **(16)** shows that if $c_i\geq 0$, then c_r is necessarily zero, showing thereby that modes, whether neutral or unstable must be non-oscillatory. Hence, it is clear that oscillatory modes are, stable.

It is to be noted that due to the presence of porous media, we are not able to characterize the stable modes in general. As it is clear from equations **(15)** and **(16)**, that $c_i<0$, we can say nothing about c_r and hence about the character of stable modes.

Theorem 3.2 : If $N^2>0$ everywhere in the flow domain, then the system is stable.

Proof : Let if possible, the system be unstable when $N^2 > 0$. Then $c_i > 0$ and $c_r = 0$ in view of the theorem (1). Equation **(15)** now becomes

$$\int \rho\Big[\Big\{c_i^2(1+k_1^{-1}R_{ve}^{-1}) - \frac{(1+k_1^{-1}R_{ve}^{-1})(k^{-1}R_D^{-1}c_i^3 + (1+k_1^{-1}R_{ve}^{-1})c_i^4)}{c_0^2} + k^{-1}R_D^{-1}\left(c_i + \frac{R_D^{-1}k^{-1}c_i^2 + (1+k_1^{-1}R_{ve}^{-1})c_i^3}{c_0^2}\right)\Big\}\frac{1}{|\Delta|^2} - S \Big]|Dw|^2\,dz + k^2\int \rho\{c_i^2(1+k_1^{-1}R_{ve}^{-1}) + R_D^{-1}k^{-1}c_i + S\}|w|^2\,dz + \int N^2\rho|w|^2\,dz = 0 \quad \textbf{(17)}$$

which is mathematically inconsistent. Hence, it follows that the system must be stable under the condition $N^2 > 0$ everywhere in the flow domain.

Corollary : If $S < \frac{1}{\pi^2}|N|^2$ and $k^2 > \frac{|N|^2}{S} - \pi^2$ everywhere in the flow domain, then the system is stable.

Theorem 3.3 : An upper bound on c_i associated with an arbitrary unstable mode if exists when $N^2 < 0$ is given by

$$c_i < \frac{|N|^2_{\max} - k^2 S_{\min}}{kR_D^{-1}}.$$

Proof : Multiplying equation **(14)** by c^* and separating its imaginary part, we have

$$\int \rho\left[c_i + (1+k_1^{-1}R_{ve}^{-1})\,.\,\frac{\{(1+k_1^{-1}R_{ve}^{-1})|c|^4 + k^{-1}R_D^{-1}c_i|c|^2\}}{c_0^2} + R_D^{-1}k^{-1}\left\{1 - \frac{(1+k_1^{-1}R_{ve}^{-1})(c_r^2 - c_i^2) - k^{-1}R_D^{-1}c_i}{c_0^2}\right\}\right]\frac{|c|^2}{|\Delta|^2}|Dw|^2\,dz + \int \rho c_i S|Dw|^2\,dz + \int \rho k^2\{c_i(1+k_1^{-1}R_{ve}^{-1}) + k^{-1}R_D^{-1}\}|c|^2|w|^2\,dz + \int \rho k^2 c_i S|w|^2\,dz + \int c_i|N|^2\rho|w|^2\,dz = 0\cdot \quad \textbf{(18)}$$

For $N^2 < 0$, rearrangement of terms in equation **(18)** yields

$$\int \rho\left[c_i + (1+k_1^{-1}R_{ve}^{-1})\,.\,\frac{\{(1+k_1^{-1}R_{ve}^{-1})|c|^4 + k^{-1}R_D^{-1}c_i|c|^2\}}{c_0^2} + R_D^{-1}k^{-1}\left\{1 - \frac{(1+k_1^{-1}R_{ve}^{-1})(c_r^2 - c_i^2) - k^{-1}R_D^{-1}c_i}{c_0^2}\right\}\right]\frac{|c|^2}{|\Delta|^2}|Dw|^2\,dz + \int \rho c_i S|Dw|^2\,dz + \int \rho k^2\{c_i(1+k_1^{-1}R_{ve}^{-1})\}|c|^2|w|^2\,dz + \int \rho\{R_D^{-1}k|c|^2 + (Sk^2 - |N|^2)c_i\}|w|^2\,dz = 0\cdot \quad \textbf{(19)}$$

Let the system be unstable. Then equation **(19)** in view of theorem (1) is consistent if $R_D^{-1}kc_i^2 + (Sk^2 - |N|^2)c_i < 0$ somewhere in the flow domain

Comparing the above bounds for unstable modes with those obtained by Agrawal and Rastogi **[17]**, we see that porous medium has a stabilizing effect in view of the observation that the bounds on c_i reduce as the Darcy resistance increases.

THE CASE OF A WEAK APPLIED MAGNETIC FIELD (i.e., $S << 1$)

It is well known [Chandrasekhar **[19]**] that a suitable magnetic field tends to suppress the instability of various flows and in many cases it makes an otherwise unstable flow completely stable to small perturbations. For this reason, many authors have investigated the effect of a magnetic field on various unstable flows. Our purpose is also to see the effect of magnetic field on the stability of compressible fluid flowing through a porous medium by restricting our analysis, due to the complex nature of the problem under consideration, to weak applied magnetic field.

When $S << 1$ [Agrawal and Agrawal **[20]**], $SD^2 w$ and SDw are small enough and therefore can be neglected in comparison to $k^2 Sw$ (since w is a perturbation and their derivatives are small. It is in fact true when k is large). The final governing equation **(11)** reduces to

$$k^2 \rho\left(c^2 + ik^{-1}R_D^{-1}c + k_1^{-1}R_{ve}^{-1}c^2 - S\right) w = D\left\{\rho\left(\frac{c^2 + ik^{-1}R_D^{-1}c + k_1^{-1}R_{ve}^{-1}c^2}{\Delta}\right)\left(D - \frac{g}{c_0^2}\right)w\right\}$$
$$+ \frac{g\rho}{c_0^2}\left(c^2 + ik^{-1}R_D^{-1}c + k_1^{-1}R_{ve}^{-1}c^2\right)\left(D - \frac{g}{c_0^2}\right)w + N^2 \rho w \cdot \quad \textbf{(20)}$$

Let us now apply the transformation

$$q = wE \cdot \quad \textbf{(21)}$$

where, $$E = \exp \int \left(\frac{g}{c_0^2}\right) dz \cdot \quad \textbf{(22)}$$

Equation **(20)** now reduces to

$$\left(c^2 + ik^{-1}R_D^{-1}c + k_1^{-1}R_{ve}^{-1}c^2\right) D\left[\frac{RDq}{\Delta}\right] - k^2 R\left(c^2 + ik^{-1}R_D^{-1}c + k_1^{-1}R_{ve}^{-1}c^2\right) q$$
$$+ R(Sk^2 + N^2) q = 0 \cdot \quad \textbf{(23)}$$

where, $$R = \frac{\rho}{E^2} \quad \textbf{(24)}$$

together with the boundary conditions

$$q = 0 \text{ at } z = 0 \text{ and } z = 1. \quad \textbf{(25)}$$

Multiplying equation **(23)** by q^*, the complex conjugate of q, integrating over the range of z and using the boundary conditions, we get

$$\int R\left(\frac{c^2 + ik^{-1}R_D^{-1}c + k_1^{-1}R_{ve}^{-1}c^2}{\Delta}\right)|Dq|^2\, dz + k^2 \int R\left(c^2 + ik^{-1}R_D^{-1}c + k_1^{-1}R_{ve}^{-1}c^2\right)|q|^2\, dz$$
$$- \int R(Sk^2 + N^2)|q|^2\, dz = 0 \cdot \quad \textbf{(26)}$$

Real and imaginary parts of equation **(26)** are respectively given by

$$\int R\Bigg[\left(c_r^2 - c_i^2\right)(1 + k_1^{-1}R_{ve}^{-1}) - \frac{(1 + k_1^{-1}R_{ve}^{-1})(k^{-1}R_D^{-1}c_i\,|c|^2 + (1 + k_1^{-1}R_{ve}^{-1})|c|^4)}{c_0^2}$$
$$- k^{-1}R_D^{-1}\left\{c_i + \frac{R_D^{-1}k^{-1}|c|^2 + (1 + k_1^{-1}R_{ve}^{-1})c_i\,|c|^2}{c_0^2}\right\}\Bigg]\frac{1}{|\Delta|^2}|Dq|^2\, dz$$
$$+ k^2 \int R\left\{(c_r^2 - c_i^2)(1 + k_1^{-1}R_{ve}^{-1}) - k^{-1}R_D^{-1}c_i\right\}|q|^2\, dz$$
$$- \int R(Sk^2 + N^2)|q|^2\, dz = 0 \quad \textbf{(27)}$$

and $$c_r\left\{2c_i(1 + k_1^{-1}R_{ve}^{-1}) + k^{-1}R_D^{-1}\right\}\int \rho\left(\frac{|Dq|^2}{|\Delta|^2} + k^2|q|^2\right)dz \cdot \quad \textbf{(28)}$$

We now prove the following theorems :

Theorem 4.1 : Neutral or unstable modes are non-oscillatory.
Proof : Proof immediately from equation **(28)**. By implication, it follows that the oscillatory modes are stable.

Theorem 4.2 : If $N^2 > 0$ everywhere in the flow domain, the system is stable.

Proof : For $N^2 > 0$, let the system be unstable, i.e., $c_i > 0$, then in view of theorem (1), c_r should be zero. But equation **(27)** becomes mathematically inconsistent on substituting $c_r = 0$ when $N^2 > 0$. Therefore c_i can not be greater than zero. The similar remark is germane if we start with $c_i = 0$. Thus for $N^2 > 0$ everywhere in the flow domain, $c_i > 0$ necessarily implying thereby the stability of the system.

We observe that this result holds true even in the absence of a magnetic field(i.e., $S = 0$). In fact, we are more interested in the case when $N^2 < 0$, in order to demonstrate the stabilizing role of magnetic field and porous medium. $N^2 < 0$ is the case when either $D\rho > 0$ (statically unstable arrangement) or if $D\rho < 0$, then $\left|\frac{D\rho}{\rho}\right| < \frac{g}{c_0^2}$ having a possibility of the unstable modes to occur in the absence of magnetic field. Therefore, in the subsequent theorems, we will discuss the case when $N^2 < 0$.

Theorem 4.3 : If $N^2 < 0$ and $k^2 \geq |N|^2/S$ everywhere in the flow domain, the system is stable.
Proof : For $N^2 < 0$, equation **(27)** can be written as

$$\begin{aligned}\int R\Bigg[&\left(c_r^2 - c_i^2\right)(1+k_1^{-1}R_{ve}^{-1}) - \frac{(1+k_1^{-1}R_{ve}^{-1})(k^{-1}R_D^{-1}c_i\,|c|^2 + (1+k_1^{-1}R_{ve}^{-1})\,|c|^4)}{c_0^2}\\ &- k^{-1}R_D^{-1}\left\{c_i + \frac{R_D^{-1}k^{-1}\,|c|^2 + (1+k_1^{-1}R_{ve}^{-1})c_i\,|c|^2}{c_0^2}\right\}\Bigg]\frac{1}{|\Delta|^2}\,|Dq|^2\,dz\\ &+ k^2\int R\left\{(c_r^2 - c_i^2)(1+k_1^{-1}R_{ve}^{-1}) - k^{-1}R_D^{-1}c_i\right\}|q|^2\,dz\\ &- \int R(Sk^2 - |N|^2)\,|q|^2\,dz = 0\,. \end{aligned} \qquad \textbf{(29)}$$

Theorem (1) shows that if $c_i \geq 0$, equation **(29)** becomes inconsistent if the condition

$$k^2 \geq \frac{|N|^2}{S} \qquad \textbf{(30)}$$

holds everywhere in the flow domain. It follows that the system is stable for the wave numbers given by **(30)** if $N^2 < 0$.

It is clear from the above theorem that the range of stable wave numbers increases as magnetic field S increases. This establishes the stabilizing character of magnetic field in the present context.

It follows from the above discussion that instability might occur for small wave numbers in the range $0 < k^2 < \frac{|N|^2}{S}$.

In the next theorem, therefore, we have made an attempt to find the bounds on c_i associated with arbitrary unstable modes, if exist in the wave number range of $0 < k^2 < \frac{|N|^2}{S}$.

ANALYSIS FOR SUBSONIC DISTURBANCES

This section deals with the hydromagnetic stability of subsonic disturbances in the presence of an arbitrary magnetic field. The stability governing equation **(11)** for subsonic disturbances reduces to

$$\begin{aligned}k^2\rho(c^2 + ik^{-1}R_D^{-1}c + k_1^{-1}R_{ve}^{-1}c^2 - S)w = D\Bigg\{&\rho\left(\frac{c^2 + ik^{-1}R_D^{-1}c + k_1^{-1}R_{ve}^{-1}c^2}{\Delta_1} - S\right)Dw\\ &- \frac{g\rho}{c_0^2\Delta_1}(c^2 + ik^{-1}R_D^{-1}c + k_1^{-1}R_{ve}^{-1}c^2)w\Bigg\}\end{aligned}$$

$$+\frac{g\rho}{c_0^2\Delta_1}(c^2+ik^{-1}R_D^{-1}c+k_1^{-1}R_{ve}^{-1}c^2)\left(D-\frac{g}{c_0^2}\right)w+N^2\rho w\cdot \qquad \textbf{(31)}$$

Multiplying equation **(31)** by w^* integrating over the range of z and using the boundary conditions, we get

$$\int \rho\left(\frac{c^2+ik^{-1}R_D^{-1}c+k_1^{-1}R_{ve}^{-1}c^2}{\Delta_1}-S\right)|Dw|^2\,dz$$
$$+k^2\int \rho(c^2+ik^{-1}R_D^{-1}c+k_1^{-1}R_{ve}^{-1}c^2-S)|w|^2\,dz$$
$$+\int \rho\left(\frac{g}{c_0^2}\right)^2\left(\frac{c^2+ik^{-1}R_D^{-1}c+k_1^{-1}R_{ve}^{-1}c^2}{\Delta_1}\right)|w|^2\,dz-\int N^2\rho|w|^2\,dz=0\cdot \qquad \textbf{(32)}$$

The real and imaginary parts of equation **(32)** are respectively given by

$$\int \rho\left[\left\{(c_r^2-c_i^2)(1+k_1^{-1}R_{ve}^{-1})-\frac{k^{-1}R_D^{-1}(1+k_1^{-1}R_{ve}^{-1})c_i\,|c|^2}{c_0^2}\right.\right.$$
$$\left.\left.-k^{-1}R_D^{-1}\left(c_i+\frac{k^{-1}R_D^{-1}|c|^2}{c_0^2}\right)\right\}\frac{1}{|\Delta_1|^2}-S\right]|Dw|^2\,dz$$
$$+\int g\left\{(c_r^2-c_i^2)(1+k_1^{-1}R_{ve}^{-1})-\frac{k^{-1}R_D^{-1}(1+k_1^{-1}R_{ve}^{-1})c_i\,|c|^2}{c_0^2}\right.$$
$$\left.-k^{-1}R_D^{-1}\left(c_i+\frac{k^{-1}R_D^{-1}|c|^2}{c_0^2}\right)\right\}\frac{1}{|\Delta_1|^2}\left\{D\left(\frac{\rho}{c_0^2}\right)\right\}|w|^2\,dz$$
$$+\int \left(\frac{g}{c_0^2}\right)^2\rho\left\{(c_r^2-c_i^2)(1+k_1^{-1}R_{ve}^{-1})-\frac{k^{-1}R_D^{-1}(1+k_1^{-1}R_{ve}^{-1})c_i\,|c|^2}{c_0^2}\right.$$
$$\left.-k^{-1}R_D^{-1}\left(c_i+\frac{k^{-1}R_D^{-1}|c|^2}{c_0^2}\right)\right\}\frac{|w|^2}{|\Delta_1|^2}\,dz$$
$$+k^2\int \rho\left\{(c_r^2-c_i^2)(1+k_1^{-1}R_{ve}^{-1})-k^{-1}R_D^{-1}c_i-S\right\}|w|^2\,dz-\int N^2\rho|w|^2\,dz=0 \qquad \textbf{(33)}$$

and

$$c_r\left(2c_i(1+k_1^{-1}R_{ve}^{-1})+k^{-1}R_D^{-1}\right)\left\{\int \rho\frac{|Dw|^2}{|\Delta_1|^2}\,dz+\int \frac{g}{|\Delta_1|^2}\left\{D\left(\frac{\rho}{c_0^2}\right)\right\}|w|^2\,dz\right.$$
$$\left.+k^2\int \rho|w|^2\,dz+\int\left(\frac{g}{c_0^2}\right)^2\rho\frac{|w|^2}{|\Delta_1|^2}\,dz\right]=0\cdot \qquad \textbf{(34)}$$

We now prove some theorems given below

Theorem 5.1 : Neutral or unstable modes are non-oscillatory under the condition $D(\rho/c_0^2)>0$.

Theorem 5.2 : If $N^2>0$ and $D(\rho/c_0^2)>0$, everywhere in the flow domain, the system is stable.

Theorem 5.3 : If $N^2<0,\ D(\rho/c_0^2)>0$ and $k^2\geq|N|^2/S$, the system is stable.

Theorem 5.4 : If $N^2<0$ and $D(\rho/c_0^2)>0$ everywhere in the flow domain, then an upper bound on the complex wave velocity of arbitrary unstable mode, if exists, is given by

$$c_i<\frac{|N|_{\max}^2-k^2S_{\min}}{kR_D^{-1}}\cdot$$

Proof : The imaginary part of equation **(31)** after multiplying it by c^* is given by

$$\int \left\{c_i(1+k_1^{-1}R_{ve}^{-1})+\frac{(1+k_1^{-1}R_{ve}^{-1})k^{-1}R_D^{-1}|c|^2}{c_0^2}+k^{-1}R_D^{-1}\left(1+\frac{k^{-1}R_D^{-1}c_i}{c_0^2}\right)\right\}\frac{|c|^2}{|\Delta_1|^2}\left[\rho|Dw|^2\right.$$

$$+\left\{gD\left(\frac{\rho}{c_0^2}\right)+\rho\left(\frac{g}{c_0^2}\right)^2\right\}|w|^2\Bigg]dz+\int \rho Sc_i\,|Dw|^2\,dz$$

$$+k^2\int \rho\left[\left\{c_i(1+k_1^{-1}R_{ve}^{-1})+k^{-1}R_D^{-1}\right\}(|c|^2+Sc_i)\right]|w|^2\,dz$$

$$+\int N^2 c_i\rho|w|^2\,dz=0\cdot \qquad \textbf{(35)}$$

Or after some rearrangement of terms, it becomes

$$\int\left\{c_i(1+k_1^{-1}R_{ve}^{-1})+\frac{(1+k_1^{-1}R_{ve}^{-1})k^{-1}R_D^{-1}|c|^2}{c_0^2}+k^{-1}R_D^{-1}\left(1+\frac{k^{-1}R_D^{-1}c_i}{c_0^2}\right)\right\}\frac{|c|^2}{|\Delta_1|^2}\Big[\rho|Dw|^2$$

$$+\left\{gD\left(\frac{\rho}{c_0^2}\right)+\rho\left(\frac{g}{c_0^2}\right)^2\right\}|w|^2\Bigg]dz+\int \rho Sc_i\,|Dw|^2\,dz$$

$$+k^2\int \rho\left\{c_i(1+k_1^{-1}R_{ve}^{-1})\right\}|c|^2|w|^2\,dz$$

$$+\int \rho\left[kR_D^{-1}|c|^2+(Sk^2-|N|^2)c_i\right]|w|^2\,dz=0\cdot \qquad \textbf{(36)}$$

Now, let the modes be unstable under the conditions

$N^2<0$ and $D(\rho/c_0^2)>0$ everywhere in the flow domain,

then for the consistency of equation **(36)**, we must necessarily have

$kR_D^{-1}|c|^2+(Sk^2-|N|^2)c_i<0$ somewhere in the flow domain.

In view of domain of theorem (1), it follows that an estimate on c_i associated with an arbitrary unstable mode is given by

$$c_i<\frac{|N|_{\max}^2-k^2S_{\min}}{kR_D^{-1}}\cdot$$

ANALYSIS FOR SUPERSONIC DISTURBANCES

A disturbance is said to be supersonic [Lees and Lin **[21]**, Lesson, Fox and Zien **[22]** and Blumen **[23]**, etc.] iff its wave speed relative to the flow velocity in the direction of wave propagation is greater than the local sonic velocity. Therefore, for supersonic disturbances, we must have

$$1<<|(U-c)/c_0|^2, \qquad \textbf{(37)}$$

where, U is the flow velocity. In the absence of any flow velocity, condition **(37)** reduces to

$$1<<|c/c_0|^2\cdot \qquad \textbf{(38)}$$

In the present context, for supersonic disturbances, we have

$$1<<\left|\frac{c^2+ik^{-1}R_D^{-1}c+k_1^{-1}R_{ve}^{-1}c^2}{c_0^2}\right|,$$

so that

$$\Delta\approx\Delta_2=-\frac{c^2+ik^{-1}R_D^{-1}c+k_1^{-1}R_{ve}^{-1}c^2}{c_0^2}\cdot \qquad \textbf{(39)}$$

Thus, under this approximation, the stability governing equation **(11)** reduces to

$$D\left\{\rho(c_0^2+S)Dw\right\}+k^2\rho\left\{c^2(1+k_1^{-1}R_{ve}^{-1})+ik^{-1}R_D^{-1}c\right\}w=0\cdot \qquad \textbf{(40)}$$

Multiplying equation **(40)** by w^*, the complex conjugate of w, integrating over the range of z and using boundary conditions, we get

$$\int \rho(c_0^2+S)|Dw|^2\,dz-k^2\int \rho\left\{c^2\left(1+k_1^{-1}R_{ve}^{-1}\right)+ik^{-1}R_D^{-1}c-S\right\}|w|^2\,dz=0\cdot$$

(41)

The real and imaginary parts of equation **(41)** are given by

$$\int \rho(c_0^2+S)|Dw|^2\,dz-k^2\int \rho\left\{(c_r^2-c_i^2)\left(1+k_1^{-1}R_{ve}^{-1}\right)-k^{-1}R_D^{-1}c_i-S\right\}|w|^2\,dz=0$$

(42)

and $$-k^2 c_r \int (2c_i + k^{-1} R_D^{-1})\, \rho\, |w|\ dz = 0 \cdot \qquad (43)$$

We now prove the following theorem :

Theorem 6. 1 : The system is stable.

Proof : Let the system be unstable or neutral, i.e., $c_i \geq 0$, then in view of theorem (1), $c_r = 0$. But for $c_i \geq 0$ and $c_r = 0$, equation **(42)** is mathematically inconsistent. Therefore, there exists not even a single mode for which the system is unstable or neutral or in other words the system is stable.

THE GENERAL CASE

An attempt has been made in section, to discuss the problem in its full generality and following results are obtained.

Theorem 7.1 : For $N^2 < 0$, and $k^2 > \frac{|N|^2}{S}$, unstable modes, if exist, are oscillatory and the non-oscillatory modes, if exist, are stable.

Proof : After some simplification, final stability governing equation **(11)** can be written as

$$k^2 \rho (c^2 + ik^{-1} R_D^{-1} c + k_1^{-1} R_{ve}^{-1} c^2 - S) w = D\left\{ \rho \left(\frac{c^2 (1 + k_1^{-1} R_{ve}^{-1}) + ik^{-1} R_D^{-1}}{\Delta} - S \right) Dw \right\}$$
$$- wD\left\{ \frac{g\rho}{c_0^2} \left(\frac{c^2(1 + k_1^{-1} R_{ve}^{-1}) + ik^{-1} R_D^{-1} c}{\Delta} \right) \right\}$$
$$- \left(\frac{g}{c_0^2} \right)^2 \rho \left(\frac{c^2(1 + k_1^{-1} R_{ve}^{-1}) + ik^{-1} R_D^{-1} c}{\Delta} \right) w + N^2 \rho w \cdot \qquad (44)$$

Multiplying equation **(44)** by w^*, the complex conjugate of w, integrating over the range of z and using the boundary conditions, we get

$$\int \rho \left(\frac{c^2 (1 + k_1^{-1} R_{ve}^{-1}) + ik^{-1} R_D^{-1} c}{\Delta} - S \right) |Dw|^2\ dz$$
$$+ k^2 \int \rho \left\{ c^2 (1 + k_1^{-1} R_{ve}^{-1}) + ik^{-1} R_D^{-1} c - S \right\} |w|^2\ dz$$
$$+ \int \left[D\left\{ \frac{g}{c_0^2} \rho \left(\frac{c^2 + ik^{-1} R_D^{-1} c + k_1^{-1} R_{ve}^{-1} c^2}{\Delta} \right) \right\} \right.$$
$$\left. + \left(\frac{g}{c_0^2} \right)^2 \rho \left\{ \frac{c^2 (1 + k_1^{-1} R_{ve}^{-1}) + ik^{-1} R_D^{-1} c}{\Delta} \right\} - N^2 \rho \right] |w|^2\ dz = 0 \cdot \qquad (45)$$

Under the conditions stated in the theorem, equation **(45)** becomes mathematically inconsistent in the following two situations :

[1] When $c_i \geq 0$. Therefore, for arbitrary unstable or neutral modes, c_r can not be zero or in other words, arbitrary unstable or neutral modes are oscillatory.

[2] If for $c_r = 0$, we put $c_i \geq 0$. Therefore, if non-oscillatory modes exist, c_i should necessary be less than zero or the non-oscillatory modes are stable.

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